DUSO Mathematics League 2015 - 2016

CONTEST #3.

SOLUTIONS

3 - 1. $\boxed{\mathbf{(2x+5)(x-3)}}$ The factoring is of the form $(2x+A)(x+B)$ in order to obtain a lead term of $2x^2$. Note that $AB=-15$ and $2B+A=-1$. This system can be solved to obtain $A=5$ and $B=-3$, so the factoring is $\mathbf{(2x+5)(x-3)}$.

3 - 2. $\boxed{\mathbf{677710}}$ The sum of the first n terms of an arithmetic sequence is $\frac{n}{2}(a_1+a_n)=\frac{n}{2}(5+2015)=1010n$. But what is n? Well, $2015=5+(n-1)(3)\to n=671$, so the sum is $1010\cdot 671=\mathbf{677710}$.

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3 - 3. $\boxed{(\mathbf{\frac{2}{3}},\mathbf{\frac{1}{3}})}$ Find the equation of the perpendicular bisector of the segment connecting $(2,6)$ and $(6,0)$; this line is the line of reflection. First, the midpoint of the segment connecting $(2,6)$ and $(6,0)$ is $(4,3)$. Next, the slope of the line connecting $(2,6)$ and $(6,0)$ is $\frac{-3}{2}$, so the slope of the perpendicular bisector is $\frac{2}{3}$. The equation of the perpendicular bisector is $y-3=\frac{2}{3}(x-4)\to y=\frac{2}{3}x+\frac{1}{3}$. Thus, the ordered pair (m,b) is $(\mathbf{\frac{2}{3}},\mathbf{\frac{1}{3}})$.

3 - 4. $\boxed{\mathbf{3}}$ Extend $\overline{MU}$ to T on $\overline{GR}$, then $\triangle GUM\cong\triangle GUT$ by ASA, so $GT=11\to TR=6$. Notice that U and P are midpoints of $\overline{MT}$ and $\overline{MR}$, respectively, so $UP=\frac{1}{2}\cdot 6=\mathbf{3}$.

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3 - 5. $\boxed{\mathbf{3}}$ For a quadratic equation to have two real roots, its discriminant is positive. Thus, $25-4\cdot 2\cdot k>0\to k<3.125$. The greatest such integer k is $\mathbf{3}$.

3 - 6. $\boxed{\mathbf{36}}$ Approach the problem by cases. Suppose first that all four letters are distinct. In this case, there are $\binom{6}{2}=15$ sets of four letters. Next, suppose that there is exactly one pair among the four. In this case, there are two possible pairs, and $\binom{5}{2}=10$ ways to choose the remaining two letters, so there are 20 of these sets. The last set is the set $\{D,D,I,I\}$. There are altogether $15+20+1=\mathbf{36}$ distinct sets of four letters.

R-1. In a triangle of perimeter 2016, the three sides have measures x, $2x - 672$, and $3x - 1344$. Compute the degree measure of the greatest angle in the triangle.
R-1Sol. $\boxed{\mathbf{60}}$ Solve $x + 2x - 672 + 3x - 1344 = 2016$ to find $x = 672$, which means that all sides measure 672. The measure of each angle is **60**.

R-2. *Let N be the number you will receive.* In parallelogram $SCAM$, angles S and C differ by $N°$. If angle C is obtuse, compute the number of degrees in the measure of angle A.
R-2Sol. $\boxed{\mathbf{60}}$ Solve $A + A + N = 180 \rightarrow A = 90 - \frac{N}{2}$. Substituting, $A = \mathbf{60}$.

R-3. *Let N be the number you will receive.* Jimmy, Timmy, and Kimmy are playing a game. Their total score is N points. Timmy has the average score of the three players. Kimmy beat Jimmy by 10 points. Compute Jimmy's score.
R-3Sol. $\boxed{\mathbf{15}}$ The average score is $\frac{N}{3}$. Kimmy's score is $J + 10$ where J is Jimmy's score. Thus, $J + \frac{N}{3} + J + 10 = N \rightarrow 2J + 10 = \frac{2N}{3} \rightarrow J = \frac{N}{3} - 5$. Substituting, $J = \mathbf{15}$.

R-4. *Let N be the number you will receive.* In a room with N people, every child shakes hands with every adult once. A total of 54 handshakes take place. There are more children than adults in the room. Compute the number of children.
R-4Sol. $\boxed{\mathbf{9}}$ The equation that needs solving is $K(N - K) = 54$. Substituting, $K(15 - K) = 54$ implies that $K = \mathbf{9}$.

R-5. *Let N be the number you will receive.* Old Mother Hubbard had N children, and the difference between the ages of any two consecutive children is 2 years. The sum of their ages is 234 years. How old is the oldest child?
R-5Sol. $\boxed{\mathbf{34}}$ Substituting, we know that there are 9 children. Therefore, the middle child is age $234 \div 9 = 26$ years old. The oldest is $26 + 8 = \mathbf{34}$ years old.

Author: George Reuter - coachreu@gmail.com – Reviewer: Michael Curry - currymath@gmail.com